MA114 Summer 2018
Worksheet 17 – Exam 2 Review – 7/11/18

1. Determine whether the following series are absolutely convergent, conditionally convergent, or divergent. Show your work.

   a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2-1}{n^3+1}$,

   b) $\displaystyle\sum_{k=1}^{\infty} \frac{k \ln k}{(k+1)^3}$

   c) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n}$,

   d) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n n^2}{n!}$,

   e) $\displaystyle\sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdot \ldots \cdot (2n-1)}{2 \cdot 5 \cdot 8 \cdot \ldots \cdot (3n-1)}$,

   f) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{n^3} + \frac{1}{3^n} \right)$,

   g) $\displaystyle\sum_{k=1}^{\infty} k^5 \sqrt[3]{k^{-21}}$,

   h) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{\sqrt{n}}$.

2. a) What does the integral test tell us about $\sum_{n=1}^{\infty} |\cos n| + 1/n$?

   b) What is the limit of the convergent geometric sequence $\sum_{n=1}^{\infty} r^n$?

   c) True or false: if $\lim_{n\to\infty} a_n = 0$, then $\sum a_n$ converges.

3. Find the limit of the following sequences or show that they diverge.

   a) $a_n = \dfrac{(-1)^n}{2\sqrt{n}}$

   b) $a_n = \dfrac{4^n}{1+9^n}$

   c) $a_n = \cos\left(\dfrac{2n+1}{3n^2} \pi \right)$

   d) $a_n = \ln(n+1) - \ln(e \cdot n + 2)$

4. Find the radius and interval of convergence of each power series. Remember to check endpoints.

   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n^4+2}$

   (b) $\displaystyle\sum_{n=0}^{\infty} \frac{(x-1)^{3n+2}}{\ln(n)}$

   (c) $\displaystyle\sum_{n=0}^{\infty} \frac{(-5)^n}{n!}(x+10)^n$.

5. Find power series representations and the associated radius of convergence for $\dfrac{x^3}{5+x^2}$, $\arctan(x)$ and $\dfrac{2}{(1-x)^2}$ given that $\dfrac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ on $(-1, 1)$.

6. Find the average value of $f(x) = \dfrac{\sin(\pi/x)}{x^2}$ on $[1, 2]$.